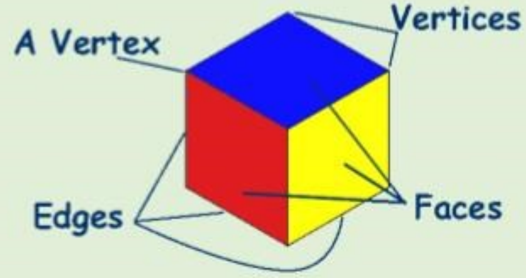
Volume and Surface Area

Maths - Higher

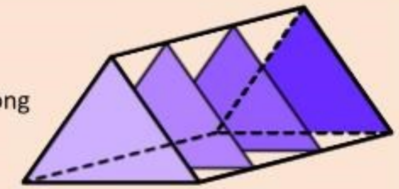
VOLUME AND SURFACE AREA – KEY WORDS AND DEFINITIONS	
Perimeter	The length around a shape.
Area	The size within a shape.
Volume	The amount of space within a 3D shape.
Surface area	The total areas of each face of a 3D shape.
Regular	All the sides and angles of a shape are equal.
Perpendicular height	The height that forms a right angle with the base length.
Face	The flat surface of a 3D shape.
Edge	The line where two faces meet.
Vertex	Where multiple edges of a 3D shape meet.
Cross section	The constant face of a prism.
Prism	A 3D shape that has the same cross-section when you cut it along its length.

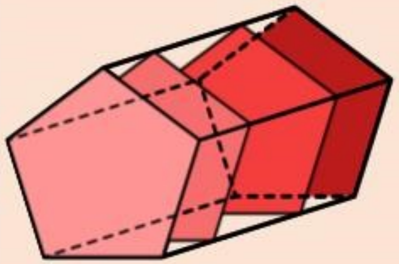
Vertices, Edges and Faces

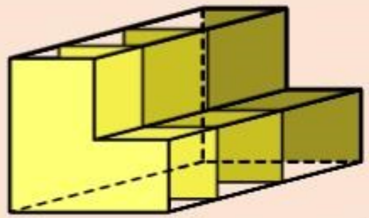


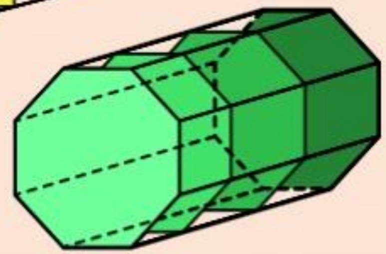
Prisms

A prism is a 3D shape that has a constant cross-section along its length.









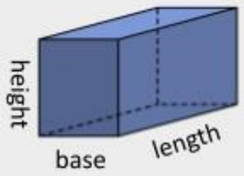
Unit Conversions

Length	Area	Volume

Maths - Higher

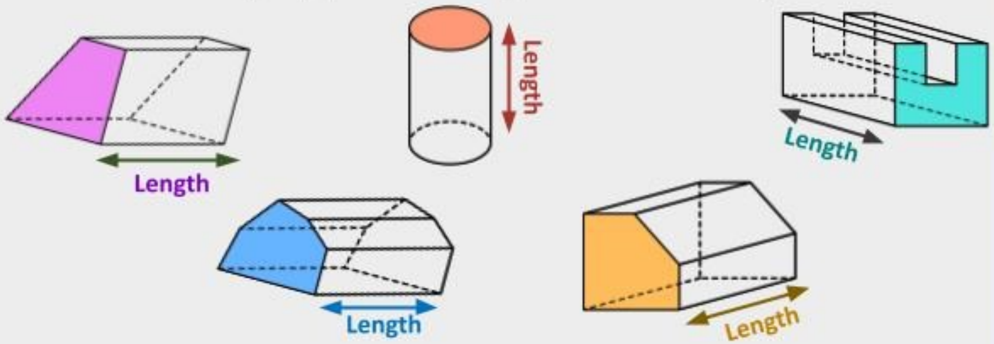
Volume and Surface Area

Volume of prisms

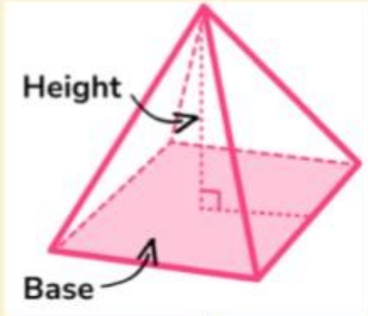


$$Volume\ of\ a\ cuboid/cube\ =\ base\ \times\ height\ \times\ length$$

$$Volume\ of\ any\ prism = area\ of\ cross\ section\ \times\ length$$



Volume of a pyramid

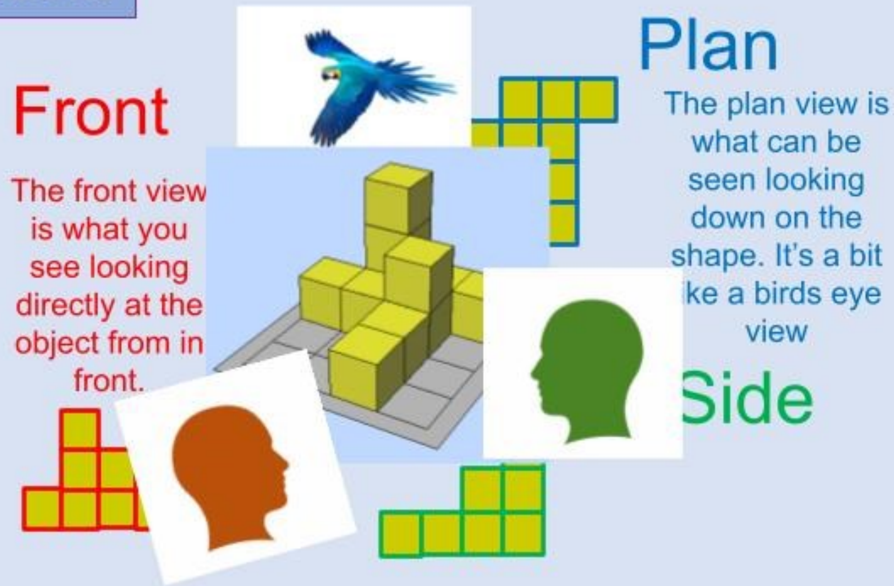


$$Volume\ = area\ of\ the\ base \times height\ \div 3$$

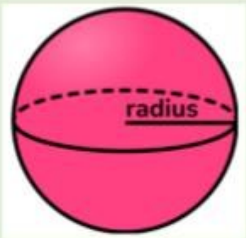
or

$$Volume = \frac{1}{3} \times area\ of\ the\ base\ \times height$$

Elevations



Volume of a sphere



$$Volume = \frac{4}{3}\pi r^3$$

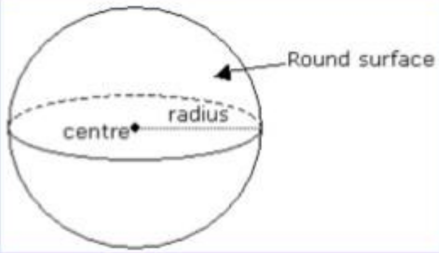
or

$$Volume = \frac{4}{3} \times \pi\ \times radius^3$$

Maths - Higher

Volume and Surface Area

Surface area of a sphere

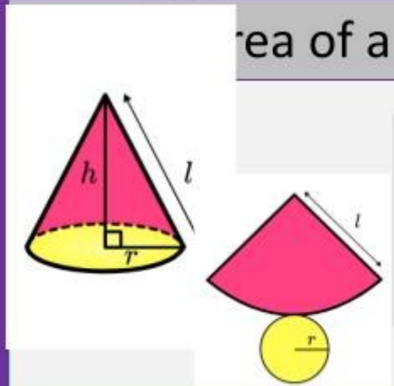


$$Surface\ area = 4\pi r^2$$

or

$$Surface\ area = 4 \times \pi \times radius^2$$

ea of a cone

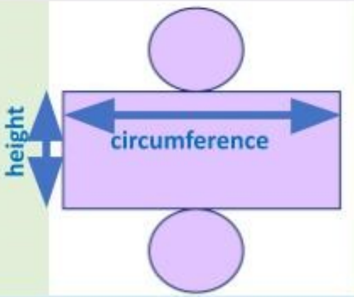


$$Surface\ area = \pi r^2 + \pi r l$$

or

$$Surface\ area = (\pi \times radius^2) + (\pi \times radius \times slant\ length)$$

Surface area of a cylinder



For a cylinder, the width of the rectangle is the circumference of the circle.

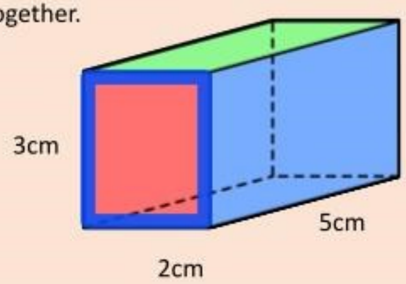
Remember:

Circumference of circle = π x diameter

Area of a circle = $\pi \times r^2$

Surface area

To find the surface area of a shape, we need to calculate the area of each face and add them altogether.



$F = 2 \times 3 = 6$

$B = 2 \times 3 = 6$

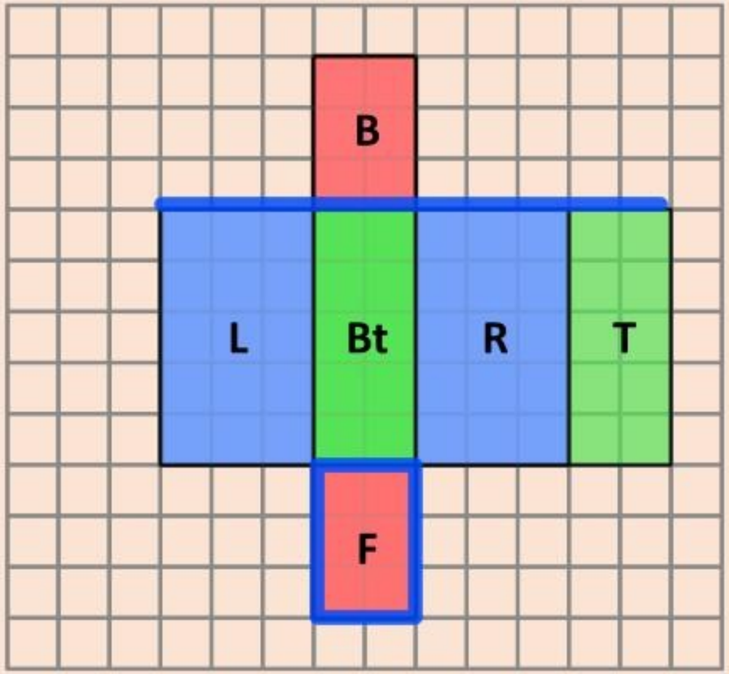
$Bt = 2 \times 5 = 10$

$T = 2 \times 5 = 10$

$L = 3 \times 5 = 15$

$R = 3 \times 5 = 15$

Total = $62cm^2$



Simultaneous equations

Maths - Higher

SIMULTANEOUS EQUATIONS – KEY WORDS AND DEFINITIONS

Simultaneous equations	A pair of equations that need to be solved at the same time. They share the same values for each of the variables.
Solution	A value we can put in place of a variable that makes the equation true.
Variable	A symbol for a number we don't know yet.
Equation	
Coefficient	
Substitute	Replace a variable with a numerical value.
LCM	Lowest common multiple. The lowest value that is in the times table of the given numbers.
Eliminate	To remove.

Steps for solving simultaneous equations

Step 1 – Rearrange your equations, if needed, so that the variables are in the same order and on the same side of the equals sign.

Step 2 – Match up the numbers in front of one of your variables. You may need to multiply one or both equations to do this. You only need to do this for one variable. It does not matter which one you choose, you will still end up with the same result at the end.

Step 3 – Add or subtract the two equations so that you eliminate the terms with the same number in front. (Same Sign Subtract, Add If Different)

Step 4 – Solve the resulting equation.

Step 5 – Substitute the result from step 4 back into one of the original equations and solve it for the remaining variable.

Solving simultaneous equations

Solve these simultaneous equations.

$$3x + 5y = 1$$
$$x - y = -5$$

Step 1 – These are already written with the variables in the same order so we do not need to do anything.

Step 2 – You have a choice now whether you make the x or y values the same. We are going to make the x value the same for this example.
To do this, we need to multiply the second equation by 3 so that both equations have $3x$.

The first equation does not need to change.

$$3x + 5y = 1$$
$$3x - 3y = -15$$

Same Sign Subtract

Step 3 – Both $3x$ are positive. $0x + 8y = 16$

Step 4 – Solve the resulting equation $8y = 16$

÷8 ÷8

$$y = 2$$

Step 5 – Substitute $y = 2$ into either of the original equations. We are going to choose the first one for this example.

$$3x + 5(2) = 1$$

$$3x + 10 = 1$$

-10 -10

$$3x = -9$$

÷3 ÷3

$$x = -3$$

By solving our equations simultaneously, we have found that $x = -3$ and $y = 2$.

These are the coordinates where the two lines intersect (-3, 2).

Maths - Higher

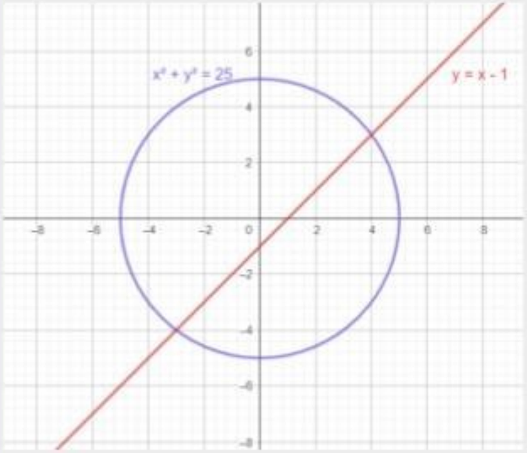
Simultaneous equations

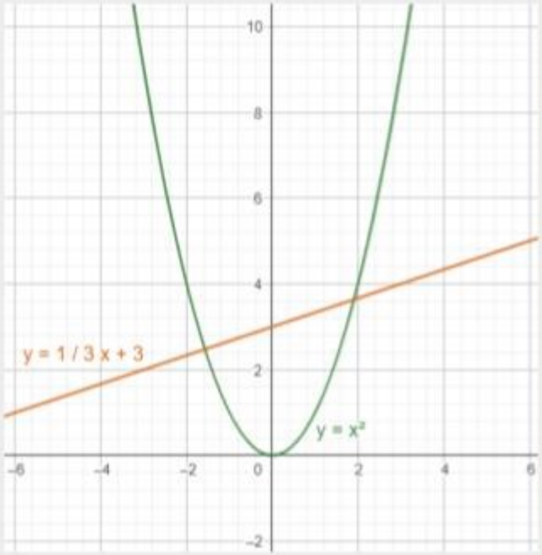
Quadratic simultaneous equations

There are a few more steps involved when it comes to solving simultaneous equations with a quadratic equation involved.

We will most likely need to solve a quadratic equation either by factorising or by using the quadratic formula.

You will end up with two values of x and y. These are the coordinates of intersection.





Solving quadratic simultaneous equations

Solve these simultaneous equations.

$$y = x - 3$$
$$x^2 + y^2 = 17$$

Step 1 – Square the equation $y = x - 3$ so that we can substitute the value of y^2.

$$y^2 = (x - 3)(x - 3)$$
$$y^2 = x^2 - 6x + 9$$

Step 2 – Replace the y^2 in the second equation with the result in the first step.

$$x^2 + (x^2 - 6x + 9) = 17$$
$$2x^2 - 6x + 9 = 17$$
$$2x^2 - 6x - 8 = 0$$

Step 3 – Solve the quadratic equation.

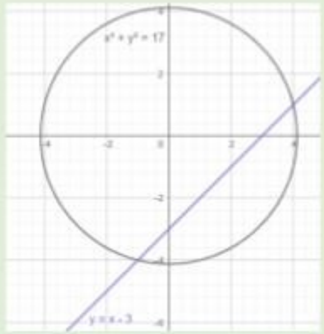
$$(2x + 2)(x - 4) = 0$$
$$x = -1 \text{ and } x = 4$$

Step 4 – Substitute the values of x back into the first equation so that we can calculate the values of y.

$$When\ x = -1, y = -1 - 3 = -4$$
$$When\ x = 4, y = 4 - 3 = 1$$

By solving our equations simultaneously we have the results:

$$x = -1 \text{ and } y = -4$$
$$x = 4 \text{ and } y = 1$$



These are the coordinates where the graphs intersect (-1, -4) and (4, 1).

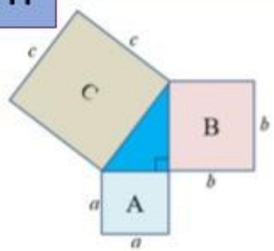
Maths - Higher

Pythagoras' Theorem

PYTHAGORAS– KEY WORDS AND DEFINITIONS	
Pythagoras' Theorem	
Hypotenuse	The longest side on a right-angled triangle. It is opposite the right angle.
Right-angled triangle	
Square number	A number that is multiplied by itself.
Square root	The inverse of squaring a number. You find the number that is squared to give you the number you have.
Unit of measure	A unit given to tell us the size of the shape. E.g. cm, m, inch, feet, etc.
Pythagorean triple	
Unit of measure	A unit given to tell us the size of the shape. E.g. cm, m, inch, feet, etc.

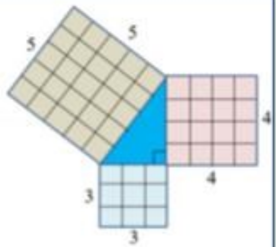
Pythagoras' Theorem

Pythagoras' theorem states that in a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.



area A + area B = area C

$a^2 + b^2 = c^2$

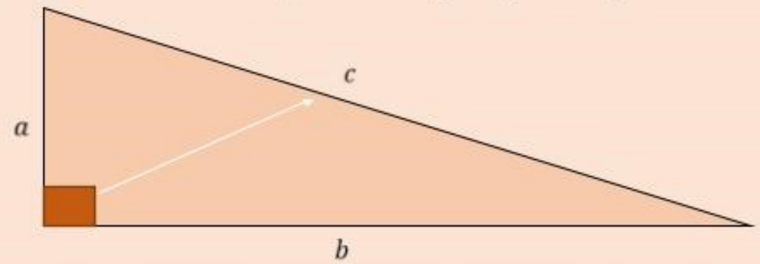


$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Labelling the triangle

Pythagoras' theorem will only work on a right angled triangle

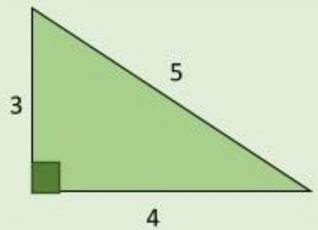


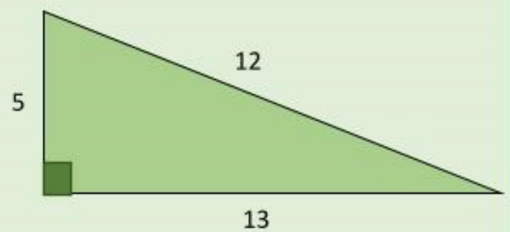
The longest side, opposite the right angle, **which we must label c**, is called the **hypotenuse**.

It does not matter which of the other two sides we label a and b.

Pythagorean Triples

Not drawn to scale





7, 24, 25

8, 15, 17

9, 40, 41

11, 60, 61

12, 35, 37

Plus many more

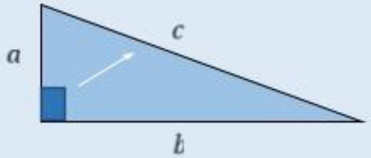
Pythagoras' Theorem

Maths - Higher

Calculating the hypotenuse, c.

We want to find the value of c.

To remove the square from c we square root.



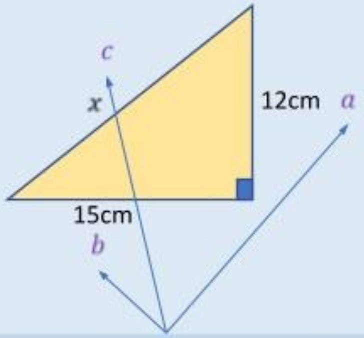
$$a^2 + b^2 = c^2$$
$$\sqrt{a^2 + b^2} = c$$

or

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$

Now we know how to calculate c, we just substitute values in for a and b.

Calculate the length of x to 3 s.f.



Label the sides with a, b and c.
Remember that c is the hypotenuse.

$$c = \sqrt{a^2 + b^2}$$

Substitute in a and b.

$$c = \sqrt{12^2 + 15^2}$$

$$c = \sqrt{144 + 225}$$

Complete the calculation in the square root.

$$c = \sqrt{369}$$

Type into your calculator.

$$c = 19.20937271$$

Round to 3 significant figures.

$$c = 19.2\ (3\ s.f.)$$

$$So\ x = 19.2cm$$

Calculating a shorter side, a or b.

How do we calculate a or b?

To calculate a:

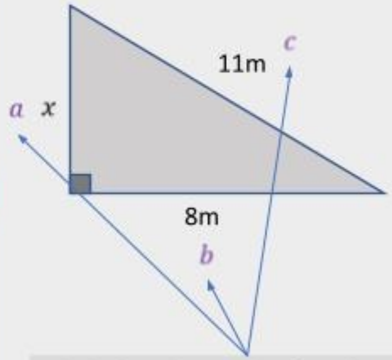
$$a^2 + b^2 = c^2$$
$-b^2$ $\quad$ $-b^2$
$$a^2 = c^2 - b^2$$
square root $\quad$ *square root*
$$a = \sqrt{c^2 - b^2}$$

To calculate b:

$$a^2 + b^2 = c^2$$
$-a^2$ $\quad$ $-a^2$
$$b^2 = c^2 - a^2$$
square root $\quad$ *square root*
$$b = \sqrt{c^2 - a^2}$$

Now we know how to calculate a or b, we just substitute the other values in.

Calculate the length of x to 2 s.f.



Label the sides with a, b and c.
Remember that c is the hypotenuse.

$$a = \sqrt{c^2 - b^2}$$

Substitute in c and b.

$$a = \sqrt{11^2 - 8^2}$$

$$a = \sqrt{121 - 64}$$

Complete the calculation in the square root.

$$a = \sqrt{57}$$

Type into your calculator.

$$a = 7.549834435$$

Round to 2 significant figures.

$$a = 7.5\ (2\ s.f.)$$

$$So\ x = 7.5m$$

Maths - Higher

Trigonometry

TRIGONOMETRY – KEY WORDS AND DEFINITIONS

Hypotenuse	The longest side in a right angled triangle.
Opposite	The side facing the angle in a right angled triangle
Adjacent	The side next to the angle given in a right angled triangle.
Square number	The result when you multiply a number by itself.
Inverse operation	The operation that reverses the effect of another operation.
Square root	The inverse operation of squaring.

When and why do we use Trigonometry?

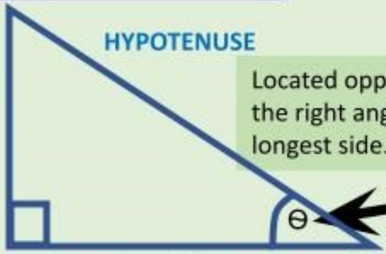
We use trigonometry to find missing lengths and angles in right angled triangles.
To find a missing side, we need to have an angle and a side.
To find a missing angle, we need to have two sides.
We also need to be able to recall exact trigonometric values for non-calculator questions.
For calculator questions, you will use the sin, cos and tan buttons on your calculator.

Labelling the triangle

OPPOSITE

Located opposite the included angle.

HYPOTENUSE



Located opposite the right angle the longest side.

Greek letter 'Theta' – represents the angle.

ADJACENT

Located between the right angle and the included angle.

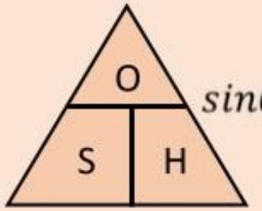
HYPOTENUSE

ADJACENT

OPPOSITE

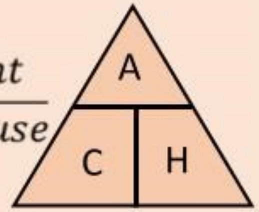
SOHCAHTOA

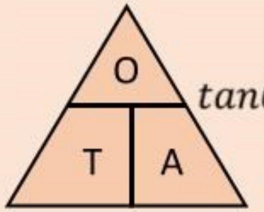
Cover over the part that you need, then complete the calculation with the remaining two.



$$sin\theta = \frac{opposite}{hypotenuse}$$

$$cos\theta = \frac{adjacent}{hypotenuse}$$



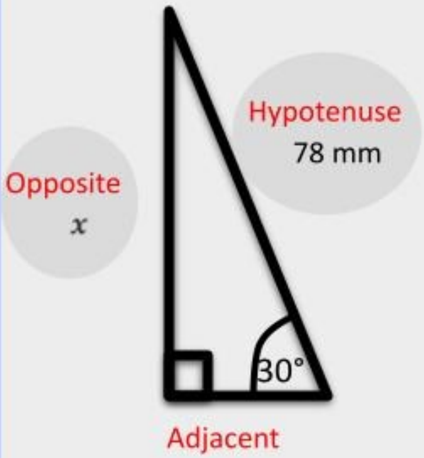


$$tan\theta = \frac{opposite}{adjacent}$$

Trigonometry

Maths - Higher

SOHCAHTOA – Missing side



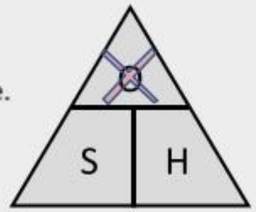
Label the sides.

Which sides have information on?

O and H.

$sin\theta$ uses O and H.

Write down the triangle.



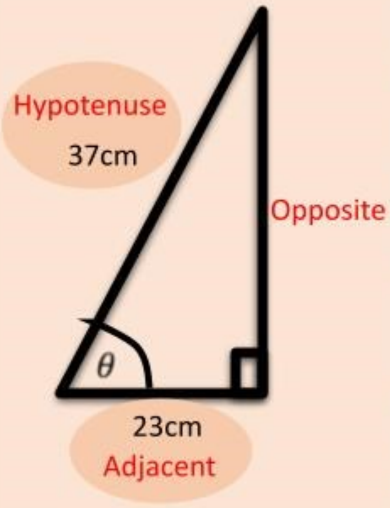
We want to cover over the O which leaves us with:

$Opposite = sin\theta \times hypotenuse$

$x = \sin(30^o) \times 78$

$x = 39mm$

SOHCAHTOA – Missing angle



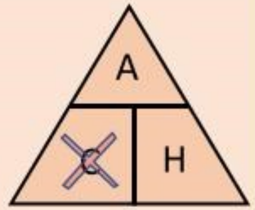
Label the sides.

Which sides have information on?

A and H.

$cos\theta$ uses A and H.

We want to cover over the cos which leaves us with:



$cos\theta = \frac{adjacent}{hypotenuse}$

$cos\theta = \frac{23}{37}$

As we want to find the angle θ, we need to do the inverse of cos to take it onto the other side.

$\theta = \cos^{-1}\left(\frac{23}{37}\right) = 52.6^o$ (1 d.p.)

Exact trigonometric values

You need to be able to remember the exact values of some trigonometric values. This will come up in a non-calculator assessment.

There are lots of ways to do this, ask your teacher to show you!

You can use triangles, hands or make your own table!

Sine	Cosine	Tangent

Maths - Higher

Trigonometry

How do I know which rule to use?

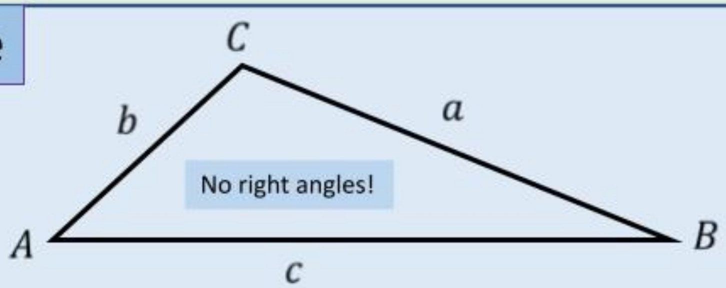
If we are asked to find lengths or angles in a triangle that is not right-angled, you will need to use either the Sine rule or the Cosine rule.

If you have a right-angled triangle and there are **no angles** involved, you need to use **Pythagoras' Theorem.**

If you have a right-angled triangle, and there **are angles** involved, you will need right-angled **Trigonometry**, using **SOHCAHTOA.**

If you have a triangle that is **not right-angled**, you will need to use the **Sine Rule** or the **Cosine Rule.** Depending on the information given, you will decide which rule is the correct one to use.

Sine Rule



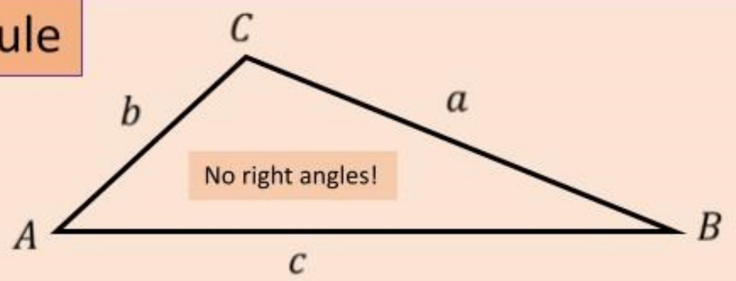
Finding an angle $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

Don't forget to use inverse sin to find the angle value.

Finding a side $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

You can only use the Sine rule if you have a "matching pair".
You have to know one angle, and the side opposite it.

Cosine Rule



If there is not a matching pair (a side and opposite angle), you will need to use the Cosine Rule.

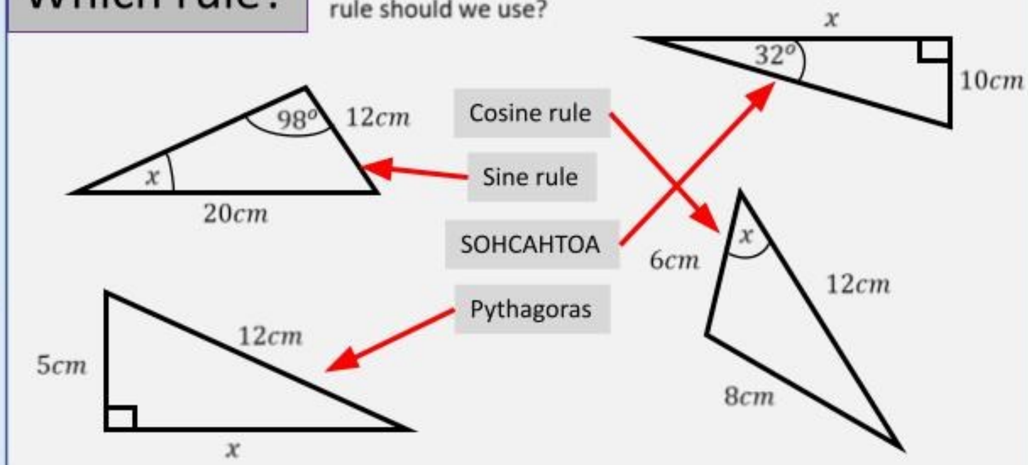
Finding a side $a^2 = b^2 + c^2 - 2bccosA$

Don't forget to square root to find the side value.

Finding an angle $A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$

Which rule?

In each triangle, we want to calculate the value of x. Which rule should we use?



Compound measures

Maths - Higher

COMPOUND MEASURES– KEY WORDS AND DEFINITIONS	
Compound measure	A measure made up of two or more measurements (e.g. speed, pressure, density)
Unit	A unit given to tell us the size of the shape. E.g. cm, m, inch, feet, etc.
Density	The amount of mass in a volume. It tells us how tightly matter is packed together.
Mass	A measure of how much matter is in an object.
Volume	The amount of 3-dimensional space an object takes up.
Pressure	The physical force exerted on an object.
Force	The push and pull of an object.
Area	The amount of space inside the boundary of a flat 2D object such as a circle or square.
Speed	How fast something is moving.
Distance	A measurement of length, how far travelled through space.
Time	Time is the ongoing sequence of events taking place. The common units of time are seconds, minutes, hours, days, weeks, months and years.

Density, Mass and Volume

Common units:

$$Density = \frac{mass}{volume}$$

g/cm^3 kg/m^3

$$Volume = \frac{mass}{density}$$

mm^3 cm^3 m^3

$$Mass = density \times volume$$

g kg

Speed, Distance and Time

Common units:

mph km/h m/s
$miles$ km m
$seconds$ $minutes$ $hours$

$$Speed = \frac{distance}{time}$$

$$Time = \frac{distance}{speed}$$

$$Distance = speed \times time$$

Pressure, Force and Area

Common units:

N/mm^2 N/cm^2 N/m^2 N mm^2 cm^2 m^2

$$Pressure = \frac{force}{area}$$

$$Force = pressure \times area$$

$$Area = \frac{force}{pressure}$$

Sequences

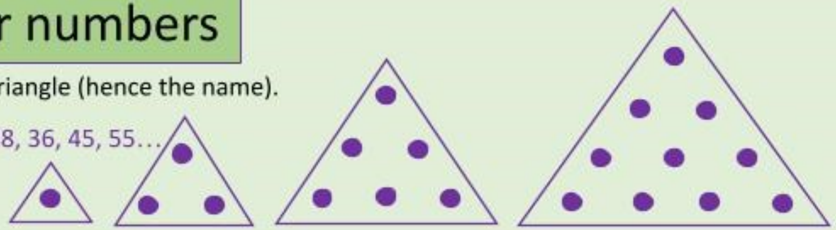
Maths - Higher

SEQUENCES – KEY WORDS AND DEFINITIONS	
Sequence	A sequence is an arrangement of objects or a set of numbers in a particular order followed by some rule.
Linear	The difference between terms increases or decreases by the **same** value each time.
Non-Linear	The difference between terms increases or decreases by **different** values each time.
Term	A single number or variable.
Position	The place a term is located.
Rule	The instructions that relate variables.
Difference	The gap between two terms.
Arithmetic	A sequence where the difference between the terms is constant. E.g. 5, 8, 11, 14, …
Geometric	A sequence where each term is found by multiplying the previous one by a fixed non-zero number. E.g. 2, 4, 8, 16, 32, …
Fibonacci	A Fibonacci sequence is created by adding the previous two terms together. E.g. 0, 1, 1, 2, 3, 5, 8, 13, …

Triangular numbers

These can make a triangle (hence the name).

1, 3, 6, 10, 15, 21, 28, 36, 45, 55…

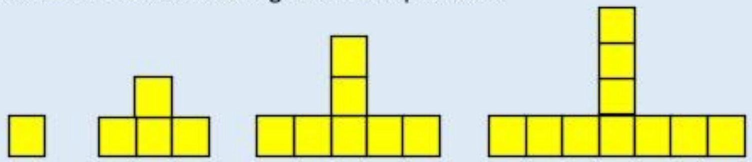


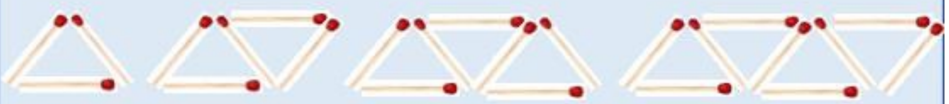
Drawing sequences

You will be given a few patterns and asked to draw the next one or two.

Look carefully at the patterns you are given – are the lines joined together or are there gaps?

How does the next one change from the previous?





Finding terms

Find the next three terms of the sequence

4, 7, 10, 13, 16…

What the sequence is increasing by

What's the difference between each term?

Add 3

don't just say "3"

The next three terms are **19, 22, 25**.

The sequence usually adds or subtracts a number between each term. If this difference keeps changing, try multiplying or dividing.

Maths - Higher

Sequences

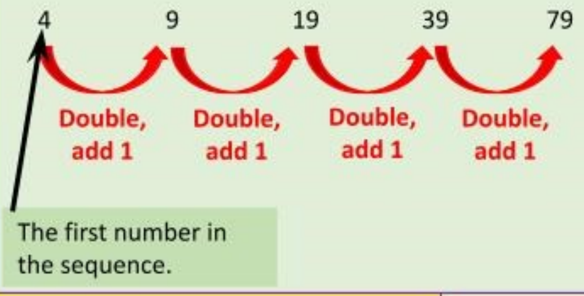
Generating sequences

To generate a sequence, you will need to follow the rule you are given.

A sequence has 4 as it's first number.

To get the next term in the sequence you "double it, then add 1".

Write the first 5 terms of the sequence.



Using the nth term

We can use the nth term to find terms in the sequence.

n is value we substitute for the term that we want to find.

The sequence "$3n - 2$" is actually "the 3 times table subtract 2".

The sequence "$2n + 1$" is "the 2 times table add 1".

The nth term for a sequence is 3n – 2. What are the first 3 terms of the sequence?

$3n - 2$ means "multiply by 3 then subtract 2".

When $n = 1, 3 \times 1 - 2 = 1$. When $n = 2, 3 \times 2 - 2 = 4$.

When $n = 3, 3 \times 3 - 2 = 7$.

So we have the first three terms 1, 4 and 7

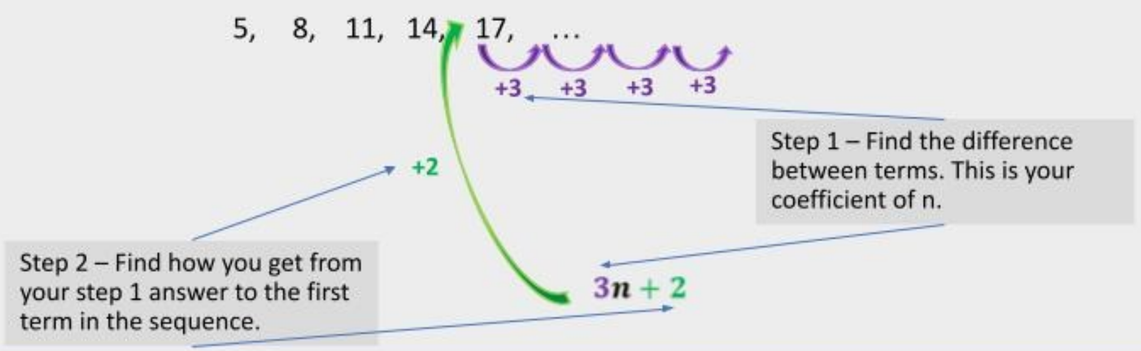
Finding the nth term – Linear Sequences

The nth term is a general formula to generate a sequence using algebra.

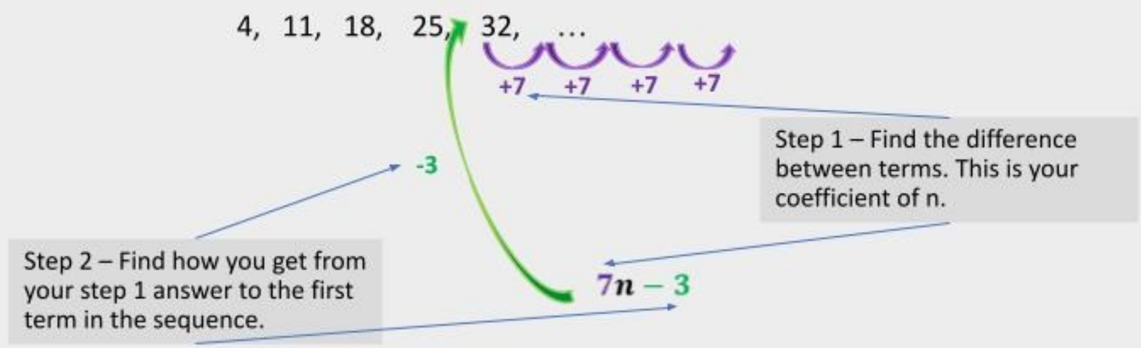
It is called the "nth term" because it contains the letter "n".

The letter "n" basically stands for "number" or the position you want in the sequence.

Find the nth term of this linear sequence.



Find the nth term of this linear sequence.



Sequences

Maths - Higher

Finding the nth term – Quadratic Sequences

When the first difference between the terms in the sequence are not the same, we know that the sequence is not linear.
We need to find the second difference. If these are the same, the sequence is quadratic.
Therefore, the nth term contains an n^2.

Find the nth term of the quadratic sequence 5, 9, 17, 29, 45, …

5, 9, 17, 29, 45, …

+4, +8, +12, +16

+4, +4, +4

$\div 2$

$2, 8, 18, 32, 50, \ldots$

$+3, +1, -1, -3, -5$

$2n^2 - 2n + 5$

Step 1 – Find the difference between terms. These are not the same so we need to find the second difference.

Step 2 – Find the second difference. This is double the coefficient of the squared term.

Step 3 – The n^2 coefficient is half of the second difference.

Step 4 – Write the first 5 terms of $2n^2$. These are the square numbers x2.

Step 5 – Find the difference between the $2n^2$ values and the original sequence.

Step 6 – Find the nth term of these differences.

Using the quadratic nth term

We can use the nth term to find terms in the sequence.

n is value we substitute for the term that we want to find.

The sequence "$n^2 + 2n - 3$" is actually "the square numbers plus the 2 times table, subtract 3".

The nth term for a sequence is $2n^2 - 4n + 2$. What are the first 3 terms of the sequence?

$2n^2 - 4n + 2$

When $n = 1$, $2 \times 1^2 - 4 \times 1 + 2 = 0$.

When $n = 2$, $2 \times 2^2 - 4 \times 2 + 2 = 2$.

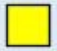
When $n = 3$, $2 \times 3^2 - 4 \times 3 + 2 = 8$.

When $n = 4$, $2 \times 4^2 - 4 \times 4 + 2 = 18$.

So we have the first four terms 0, 2, 8 and 18.

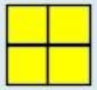
Square numbers

1^2



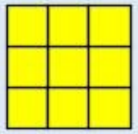
$1 \times 1 = 1$

2^2



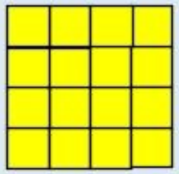
$2 \times 2 = 4$

3^2



$3 \times 3 = 9$

4^2



$4 \times 4 = 16$

5^2

$5 \times 5 = 25$

6^2

$6 \times 6 = 36$

7^2

$7 \times 7 = 49$

…