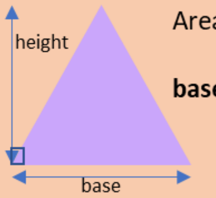
Area and Perimeter

AREA AND PERIMETER – KEY WORDS AND DEFINITIONS	
Area	The number of square units that fit inside the shape.
Perimeter	The total distance around the shape.
Compound shape	A compound shape is made from 2 or more shapes joined together.
Radius	Distance from the centre of the circle to the circumference.
Diameter	Distance from one side of the circle, through the centre, to the other side.
Circumference	The perimeter of a circle.
Dimensions	The measurements of the shape.
Unit of measure	A unit given to tell us the size of the shape. E.g. cm, m, inch, feet, etc.
Sector	A fraction of a circle. It can look like a pizza slice!
Segment	A segment is the shape created when the triangle part of a sector is removed.

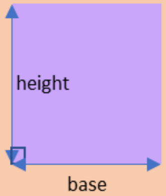
Area formulas



Area of a triangle:

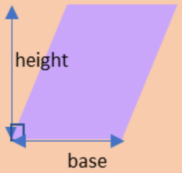
base × height ÷ 2

Height needs to be vertical!



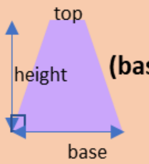
Area of a square or rectangle:

base × height



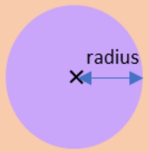
Area of a parallelogram:

base × height



Area of a trapezium:

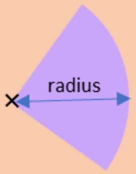
(base + top) × height ÷ 2



Area of a circle:

πr^2

where r is the radius



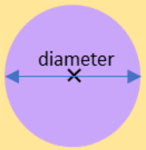
Area of a sector:

$\frac{\theta}{360} \times \pi r^2$

where r is the radius and θ is the angle.

Perimeter

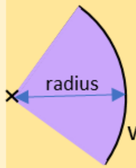
To calculate the perimeter, you find the sum of all the edges of the shape.



Circumference of a circle:

πd

where d is the diameter



Arc length of a sector:

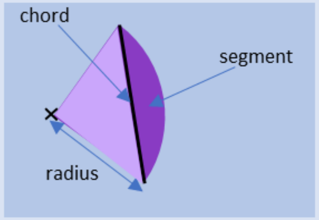
$\frac{\theta}{360} \times \pi d$

where d is the diameter and θ is the angle.

Area of a segment

To calculate the area of a segment, you find the area of the sector and subtract the area of the triangle.

For the area of the triangle, you can use $\frac{1}{2}ab\sin C$.



Area of a segment:

Area of the sector

Area of the triangle

$$\left(\frac{\theta}{360} \times \pi r^2\right) - \frac{1}{2}r^2\sin\theta$$

where r is the radius and θ is the angle.

Recurring Decimals to Fractions

RECURRING DECIMALS TO FRACTIONS – KEY WORDS AND DEFINITIONS	
Recurring decimal	A decimal that has the same figure or group of figures repeating forever e.g. 0.777777…
Terminating decimal	A decimal that ends e.g. 3.25
Simplest form	Where common factors have been taken out of the numerator and the denominator to reduce the numbers.

Examples of recurring decimals

$0.\dot{1} = 0.111111111111\ldots$

$0.1\dot{2} = 0.122222222222\ldots$

$0.\dot{1}2\dot{3} = 0.123123123123\ldots$

$0.1\dot{2}\dot{3} = 0.1232323232323\ldots$

Points to remember:

- You need to do this algebraically.
- Often this will be a proof question, so you will be given the answer and need to show the steps on how to get there.

Steps to success:

1. Let x = the recurring decimal.
2. Multiply the recurring decimal by 10, 100, 1000 until you can see the recurring part can be eliminated.
3. Subtract 1 from 2 to eliminate the recurring part.
4. Solve for x, expressing your answer as a fraction in simplest form.

EXAMPLE

One digit recurring

$0.\dot{7}$

$x = 0.777777\ldots$

$10x = 7.777777\ldots$

$10x - x = 7$

$9x = 7$

$x = \frac{7}{9}$

This part shows that you understand the notation of recurring decimals.

If you are unsure of whether to multiply by 10,100, 1000 etc, do them all and look at the figures after the decimal point and find which ones match and could be eliminated by subtraction.

EXAMPLE

Two digits recurring

$1.2\dot{5}\dot{6}$

$x = 1.2565656$

$10x = 12.565656$

$100x = 125.65656$

$1000x = 1256.5656$

$1000x - 10x = 1244$

$990x = 1244$

$x = \frac{1244}{990} = \frac{622}{495}$

Transformations

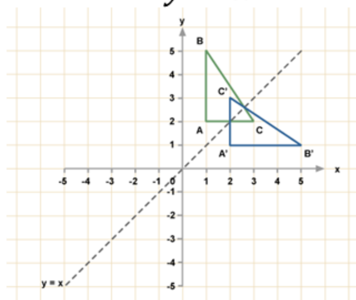
TRANSFORMATIONS – KEY WORDS AND DEFINITIONS	
Transformation	The collective name given to the four different methods of changing the position and size of a shape.
Rotation	Turns a shape on a coordinate grid from a given point. The shape does not change size but does change orientation.
Clockwise	Clockwise is the direction a clock turns. ↻
Anticlockwise	Anti-clockwise is the opposite direction. ↺
Centre	The point from where a transformation is measured
Degrees	The unit for measuring angles
Reflection	Creates a mirror image of a shape on a coordinate graph. The mirror line is given by an equation eg. $y = 2$, $x = 2$, $y = x$. The shape does not change in size.
Object	The object is the original shape.
Image	The image is the transformed shape.

Reflection

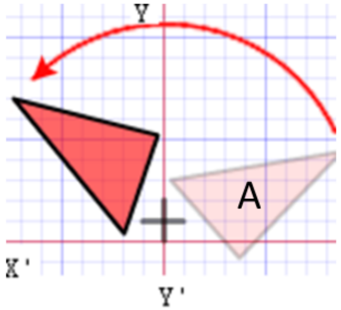
The size does not change, but the shape is 'flipped' like in a mirror.

Line $x = ?$ is a vertical line.
Line $y = ?$ is a horizontal line.
Line $y = x$ is a diagonal line.

Reflect shape C in the line
$y = x$



Rotate Shape A 90° anti-clockwise about (0,1)

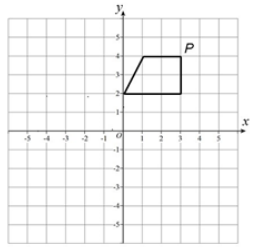


Invariance

A point, line or shape is invariant if it does not change/move when a transformation is performed.

An invariant point 'does not vary'.

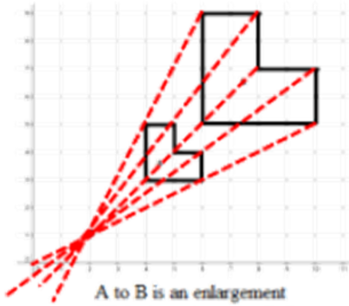
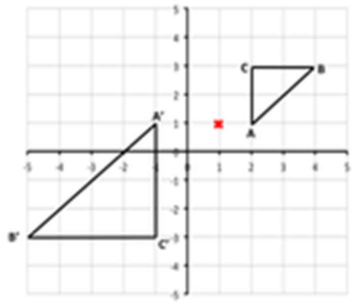
If shape P is reflected in the $y-axis$, then exactly one vertex is invariant.



Transformations

TRANSFORMATIONS – KEY WORDS AND DEFINITIONS	
Enlargement	Enlargement changes the size of an image using a scale factor from a given point.
Positive scale factor	A positive scale factor will increase the size of an image.
Fractional scale factor	A fractional scale factor will reduce the size of an image.
Negative scale factor	A negative scale factor will place the image on the opposite side of the centre of enlargement, with the image inverted.

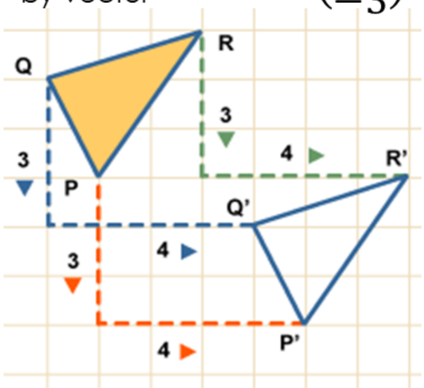
Scale Factor 3 means 3 times larger
Scale Factor ½ means half the size
Scale Factor -3 means it will be rotated and 3 times bigger

Finding the Centre of Enlargement	**Draw straight lines through corresponding corners of the two shapes.** **The centre of enlargement is the point where all the lines cross over.** **Be careful with negative enlargements as the corresponding corners will be the other way around.**	 A to B is an enlargement SF 2 about the point (2,1)
Negative Scale Factor	Negative enlargements will look like they have been rotated. $SF = -2$ will be rotated, and also twice as big.	Enlarge ABC by scale factor -2, centre (1,1) 

Transformations

TRANSFORMATIONS – KEY WORDS AND DEFINITIONS	
Translate/ Translation	To slide a shape from one position to another using a column vector. Moves a shape on a coordinate grid. Using Column vectors
Column Vector	Used to describe a translation. The top number being how many squares right (+) or left (-). The bottom number is how many square up (+) or down (-). The column vector $\binom{3}{2}$ means 3 right and 2 up. The column vector $\binom{-2}{-2}$means 2 left and 2 down.

This shape has been translated by vector $\begin{pmatrix}4\\-3\end{pmatrix}$



Positive Right $\begin{pmatrix}x\\y\end{pmatrix}$ Negative Left

Positive Up Negative Down

Describing Transformations

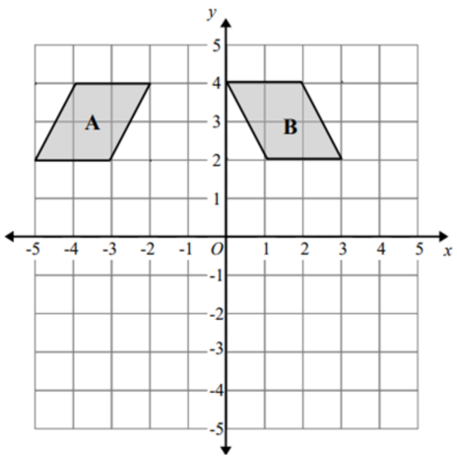
Give the following information when describing each transformation:

Look at the number of marks in the question for a hint of how many pieces of information are needed.

If you are asked to describe a 'transformation', you need to say the name of the type of transformation as well as the other details.

- **Translation: Vector**
- **Rotation: Direction, Angle, Centre**
- **Reflection: Equation of mirror line**
- **Enlargement: Scale factor, Centre of enlargement**

Describe fully the single transformation that maps shape A onto shape B.



Answer:

Reflection in the line $x = -1$

Bounds

BOUNDS – KEY WORDS AND DEFINITIONS	
Bounds	The smallest and largest values that a number could have taken before being rounded or truncated.
Error Interval/Limits of Accuracy	A range of possible values a number could have taken before it was rounded or truncated.
Truncated	A number is shortened or cut off without rounding.
Decimal Place	Position of the digits to the right of the decimal point.
Significant Figure	The first significant figure is the first non-zero digit in a number. Subsequent significant figures can include zeros.
Integer	A whole number.
Inequality	An inequality compares two values, showing if one is less than or greater than another.

For example, a mass of 70 kg, rounded to the nearest 10 kg:
The lower bound = 65kg
The upper bound = 75kg

So the error interval =
65 ≤ mass < 75

The actual mass COULD equal the lower bound

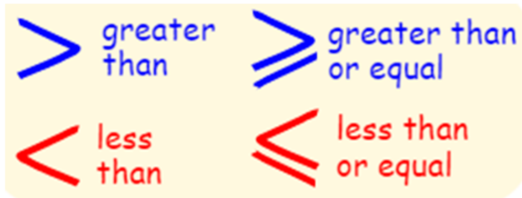
The actual mass COULDN'T equal 75 (or it would have rounded up)

General Rule:

To find the upper bound we have to add half of the unit we rounded to for example:

If we rounded to the nearest **10**, we add on **5** (half of 10)
If we rounded to the nearest **1** (whole number), we add on **0.5** (half of 1)
If we rounded to the nearest **0.1** (1 dp), we add on **0.05** (half of 0.1)
If we rounded to the nearest **0.01** (2 dp), we add on **0.005** (half of 0.01)

And if we want the lower bound, we subtract halve of the unit



a < b says that a is less than b

a ≥ b means that a is greater than or equal to b.

Bounds

Truncate 63,854 and 0.04988 to 3 significant figures.

- **63,8**|54 truncated to 3 significant figures is 63800
- **0.0498**|8 truncated to 3 significant figures is 0.0498

Bounds Calculations:
When calculating with bounds you have to start by finding the upper and lower bound of each number. Then consider which values will give you the highest possible outcome and the smallest.

Calculation	Lower bound	Upper bound
$a+b$	$a_{min}+b_{min}$	$a_{max}+b_{max}$
$a-b$	$a_{min}-a_{max}$	$a_{max}-a_{min}$
ab	$a_{min}\times b_{min}$	$a_{max}\times b_{max}$
$\frac{a}{b}$	$\frac{a_{min}}{b_{max}}$	$\frac{a_{max}}{b_{min}}$

e.g. A parallelogram has a base, *b*, of 5.64m to 2 decimal places and a perpendicular height, *h,* of 2.3m to 2 significant figures. Find the upper and lower bounds of the area, *A,* of the parallelogram.

We know this is involving bounds because it mentions rounding in the question.

The error interval of the base, b, is

$$5.635 \text{ m} \leq b < 5.645 \text{ m}$$

The error interval of the perpendicular height, h, is

$$2.25 \text{ m} \leq h < 2.35 \text{ m}$$

Lower bound of $A = 5.635 \times 2.25 = 12.67875\ m^2$

Upper bound of $A = 5.645 \times 2.35 = 13.26575\ m^2$

Quadratics

QUADRATICS – KEY WORDS AND DEFINITIONS	
Quadratic	An expression where the highest power of the variable, usually x, is squared.
Expand	To multiply each term to remove the bracket.
Factorise	The inverse of expanding. To find factors and put into brackets.
Solve	To find the value of the variable, an unknown.
Solution	A value or set of values that satisfy an equation.
Coefficient	The number which the variable is being multiplied by.
Simplify	Group and combine similar terms.
Terms	The result of a single number or variable (or numbers and variables) multiplied together.
Inverse	The opposite operation.

Factorising into a single bracket

Factorise $15x^2 + 3x$

x goes into both terms

Biggest number that goes into 15 and 3

$= 3x(5x + 1)$

What we need to multiply $3x$ by to get the original expression.

You can check your answer by expanding.

Expand Single Brackets

Expand and simplify

$3x(x - 4)$

Multiply each term in the bracket by the term on the outside.

$= 3x^2 - 12x$

Expand Double Brackets

Expand and simplify

$(x + 3)(x + 5)$

$= x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$

First
Outer
Inner
Last

Expand and simplify

$(x - 2)(x + 6)$

$= x^2 - 2x + 6x - 12$

$= x^2 + 4x - 12$

$\times$	x	-2
x	x^2	$-2x$
$+6$	$+6x$	-12

Expand and simplify

$(x + 4)^2$

$= (x + 4)(x + 4)$

$= x^2 + 4x + 4x + 16$

$= x^2 + 8x + 16$

Always write square brackets as two brackets.

Quadratics

Difference of two squares

The difference of two squares is when you have one squared term subtract another squared term.

$$a^2 - b^2 = (a+b)(a-b)$$

e.g. $y^2 - 9 = (y+3)(y-3)$

e.g. $4y^2 - 25 = (2y+5)(2y-5)$

Factorising into double brackets

Steps to factorise quadratic expressions $ax^2 + bx + c$

Step 1 – Write two brackets with x's in: $(x \quad)(x \quad)$

Step 2 – Find two numbers that multiply to give 'c' (the number term) but also add/subtract to give 'b' (the number in front of the x)

Step 3 – Put your numbers in each bracket with the correct $+$ or $-$ signs.

You can always check your answer by expanding.

Factorise $x^2 + 3x + 2$

We need to find two numbers that multiply to make 2, and add or subtract to make 3.

$$= (x+1)(x+2)$$

Factors of 2 that add to make 3.

You can check your answer by expanding.

Factorising into double brackets

Factorise $x^2 - 7x + 12$

We need to find two numbers that multiply to make 12, and add or subtract to make -7.

$$= (x-3)(x-4)$$

Factors of 12 that subtract to make -7.

Factors of 12	
1	12
2	6
3	4

You can check your answer by expanding.

Factorise $x^2 - 5x - 6$

We need to find two numbers that multiply to make -6, and add or subtract to make -5.

$$= (x-6)(x+1)$$

Factors of 6 that add and subtract to make -5.

Factors of 6	
1	6
2	3

You can check your answer by expanding.

Solving quadratics

To solve a quadratic, we need to find the value of x that makes the equation equal zero.
To do this, we factorise as normal then we find the value of x that makes each bracket 0.

Solve $x^2 + 8x - 20 = 0$

We need to find two numbers that multiply to make -20, and add or subtract to make 8.

$$(x+10)(x-2) = 0$$

To make this bracket equal zero, x must be -10.

To make this bracket equal zero, x must be +2.

So the solutions to this equation are

$$x = -10 \quad \text{and} \quad x = 2$$

Quadratics

Solving with squares

If we have an '$x^2 = $ …' where … is a number, we need to take the square root of it.

When you take the square root of a number, you will have a positive root and a negative root.

The symbol $\pm$ shows this.

Solve $4x^2 = 100$

$\div 4 \quad \div 4$

$x^2 = 25$

$\sqrt{} \quad \sqrt{}$

$x = \pm 5$

Factorise $2x^2 + 11x + 15$

Factors of 2	Factors of 15
1 2	1 15
Prime number makes it easier	3 5

One of our factors of 15 needs to be multiplied by 2, then use addition/subtraction to get +11.

Options:

- 1×1 and 2×15, 1×15 and 2×1. We know that neither of these will make 11.
- 1×3 and 2×5, 1×5 and 2×3. We know that 5 and 6 add to make 11.

So we have $(2x + 5)(x + 3)$

The pairs we multiply need to be in opposite brackets.

You can check your answer by expanding.

Factorising into double brackets when a is not 1.

Steps to factorise quadratic expressions $ax^2 + bx + c$, when $a \neq 1$

Step 1 – Write two brackets with x's in, one or both of the x's will have a number in front so leave space for this: $(\ x \quad)(\ x \quad)$

Step 2 – Find the factors of a and c.

Step 3 – Find the pair of factors from a and c that multiply together and add/subtract to give 'b' (the number in front of the x). There will be a few to try.

Step 4– Put your numbers in each bracket with the correct $+$ or $-$ signs. Your factors of a go in front of each x.

You can always check your answer by expanding.

Factorise $4x^2 - 11x + 6$

Factors of 4	Factors of 6
1 4	1 6
2 2	2 3

As 4 is not prime, we have more options to multiply and then use addition/subtraction to get -11.

Options:

1×1 and 4×6	1×2 and 4×3
1×6 and 4×1	1×3 and 4×2
2×1 and 2×6	2×2 and 2×3

We know that 3 and 8 both subtract to make -11.

So we have

$$(x + 2)(4x + 3)$$

The pairs we multiply need to be in opposite brackets.

You can check your answer by expanding.

Quadratics

The Quadratic Formula

The solutions to any quadratic expression in the form $ax^2 + bx + c = 0$ can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

All you need to do is substitute the values from your quadratic equation.

Solve $4x^2 + 8x + 3 = 0$, giving your answer to 1 d.p.

$a = 4, b = 8$ and $c = 3$

$+$ first, then go back and change to a $-$

$$x = \frac{-8 \pm \sqrt{8^2 - (4 \times 4 \times 3)}}{2 \times 4}$$

Type this into your calculator carefully, use brackets where necessary and take extra care with negatives. Know which button on your calculator is for subtract and which is for a negative number!

The solutions are $x = -0.5$ and $x = -1.5$

Completing the square when $a = 1$

Completing the square is another method we can use to solve quadratic equations, we can also use it to find the turning point of the graph.

To complete the square, you half the x coefficient and put it into squared brackets. Then you subtract the square of the number you halved and simplify.

Ask your teacher to play the completing the square song!

Express $x^2 + 4x + 3$ in the form $(x + a)^2 + b$

$\div 2$

$$(x + 2)^2 - (2)^2 + 3$$
$$(x + 2)^2 - 4 + 3$$
$$(x + 2)^2 - 1$$

Even though the general form has $+$ in it, the answer could have a $-$

Hence solve $x^2 + 4x + 3 = 0$

$$(x + 2)^2 - 1 = 0 \quad (+1)$$
$$(x + 2)^2 = 1 \quad (\sqrt{\ })$$
$$(x + 2) = \pm\sqrt{1} \quad (-2)$$
$$x = 1 - 2 \text{ and } x = -1 - 2$$
$$x = -1 \text{ and } x = -3$$

Completing the square when $a \neq 1$.

To complete the square when $a \neq 1$, we need to factorise the value of a out, complete the square as normal and then expand back out.

Express $5x^2 - 20x + 30$ in the form $a(x + b)^2 + c$

Factorise 5 out first:

$$5(x^2 - 4x + 6)$$

Now complete the square of the bracket:

$$5\left((x - 2)^2 - (-2)^2 + 6\right)$$

Simplify:

$$5\left((x - 2)^2 - 4 + 6\right)$$
$$5\left((x - 2)^2 + 2\right)$$

Expand the 5 back into the bracket:

$$5(x - 2)^2 + 5(2)$$

Simplify:

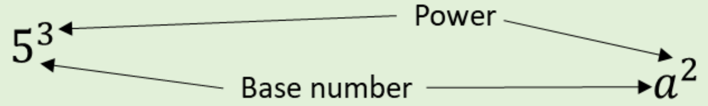
$$5(x - 2)^2 + 10$$

Indices

INDICES – KEY WORDS AND DEFINITIONS	
Index/exponent/power	An **index**, or a **power**, is the small floating number that goes next to a number or letter.
Indices	**Indices** show how many times a number or letter has been multiplied by itself.
Square number	A number or variable that has been multiplied by itself.
Cube number	A number or variable that has been multiplied by itself and then by itself again.
Square root	The **square root** of a number is the factor that we can multiply by itself to get that number.
Cube root	The **cube root** of a number is the factor that we can multiply by itself and then by itself again to get that number.
Integer	A whole number.
Coefficient	The number which the variable is being multiplied by.
Base number	The number/variable that is being multiplied by itself a given number of times.
Variable	A letter or term that represents an unknown number, value or quantity.

Powers of 10

$10^1 = 10 \quad 10^2 = 100 \quad 10^3 = 1000 \quad 10^4 = 10000$



Anything to the power of 1 is itself,
e.g. $5^1 = 5, \quad 123^1 = 123$

Anything to the power of 0 is just 1,
e.g. $6^0 = 1, \quad 4567^0 = 1$

1 to the power of anything is still 1,
e.g. $1^{10} = 1, \quad 1^{89} = 1$

$$5^3 = 5 \times 5 \times 5 = 125$$
$$a^2 = a \times a$$

Five rules to remember:

When **multiplying**, you **add** the powers.

e.g. $6^7 \times 6^4 = 6^{7+4} = 6^{11}$

When **dividing**, you **subtract** the powers.

e.g. $x^{19} \div x^{12} = x^{19-12} = x^7$

When raising one power to the another, you **multiply** the powers.

e.g. $(2^5)^8 = 2^{5\times 8} = 2^{40}$

When you have a fraction, apply the power to **both** the numerator and denominator.

e.g. $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$

A negative power turns the number **upside-down**.

e.g. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}, \quad \left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{5^2}{2^2} = \frac{25}{4}$

Square roots

Square rooting, $\sqrt{\ }$, is the inverse operation of squaring a number:

e.g. $9^2 = 9 \times 9 = 81$

$\sqrt{81} = 9$

Cube roots

Cube rooting, $\sqrt[3]{\ }$, is the inverse operation of cubing a number:

e.g. $2^3 = 2 \times 2 \times 2 = 8$

$\sqrt[3]{8} = 2$

Indices

Fractional Powers:

The **denominator** of the power tells you the **root** to take.

$$\text{e.g. } 16^{\frac{1}{2}} = \sqrt{16} = 4$$

$$\text{e.g. } 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$

$$\text{e.g. } 16^{\frac{1}{4}} = \sqrt[4]{16} = 2$$

The **numerator** of the power tells you the **power** to take. It is easier to do the root first!

$$\text{e.g. } 16^{\frac{3}{2}} = \left(\sqrt{16}\right)^3 = 4^3 = 64$$

$$\text{e.g. } 27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$$

$$\text{e.g. } 16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$$

When there is a **negative** included in the power, we need to remember to **flip the base number**.

$$\text{e.g. } 25^{-\frac{3}{2}} = \left(\frac{1}{25}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{1}}{\sqrt{25}}\right)^3 = \left(\frac{1}{5}\right)^3 = \frac{1^3}{5^3} = \frac{1}{125}$$

$$\text{e.g. } \frac{27}{64}^{-\frac{2}{3}} = \left(\frac{64}{27}\right)^{\frac{2}{3}} = \left(\frac{\sqrt[3]{64}}{\sqrt[3]{27}}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = \frac{16}{9}$$

Simplifying with indices:

Simplify each part of the question **separately**.

$$\text{e.g. } (4x^2y^5z)^3$$

Each part needs to be cubed

$$4^3 \times (x^2)^3 \times (y^5)^3 \times (z)^3$$

$$= 64x^6y^{15}z^3$$

Angles

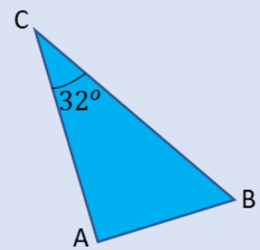
ANGLES – KEY WORDS AND DEFINITIONS	
Angle	An **angle** is formed when two lines intersect.
Acute angle	An **acute angle** is less than 90^o.
Right angle	A **right angle** is exactly $\mathbf{90^o}$.
Obtuse angle	An **obtuse angle** is greater than 90^o but less than 180^o.
Straight line	A **straight line** has an angle of $\mathbf{180^o}$.
Reflex angle	A **reflex angle** is greater than 180^o but less that 360^o.
Angles around a point.	**Angles around a point** add to make $\mathbf{360^o}$.
Parallel lines	**Parallel lines** never meet and always stay the same distance apart.
Perpendicular lines	**Perpendicular lines** intersect at 90^o.
Degree	**Degrees** are the unit measurement used to measure the size of an angle.
Interior angle	An **interior angle** is an angle on the inside of a shape.
Exterior angle	An **exterior angle** is an angle on the outside of a shape.
Polygon	A **polygon** is a many-sided shape with straight sides.
Subtended	Lines drawn from their ends form an angle at the point where they meet.

Angle notation

We use three letters to name angles. e.g. ACB = 32^o

The middle letter is where the angle is.

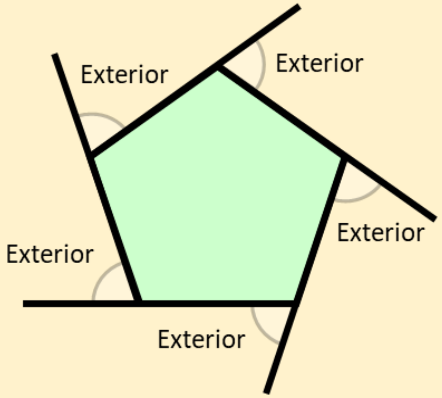
The first and last letter tell you which two lines enclose the angle.



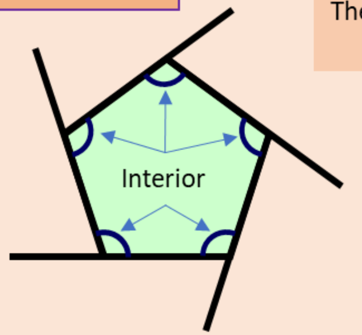
Exterior angles

Exterior angles on a polygon sum to 360^o

To calculate one exterior angle on a regular polygon:
$\mathbf{360^o}$ **÷ the number of sides**



Interior angles



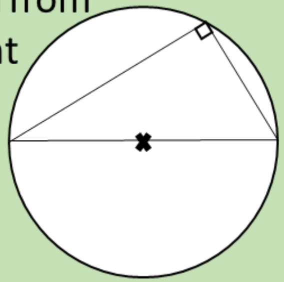
The sum of the interior angles in a polygon:
(number of sides – 2) x $\mathbf{180^o}$

To find one angle in a regular polygon, you divide the sum of the interior angles by the number of sides.

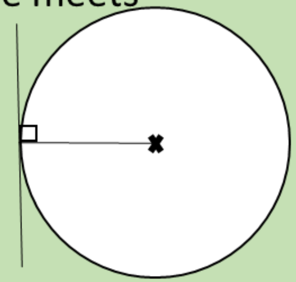
Angles

Circle Theorems

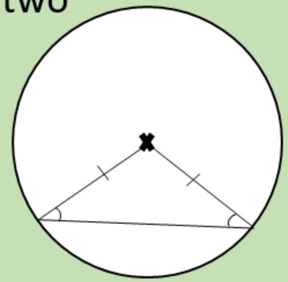
The angle subtended from the diameter is a right angle, 90^o.



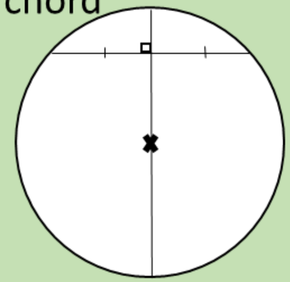
A tangent to the circle meets the radius at a right angle, 90^o.



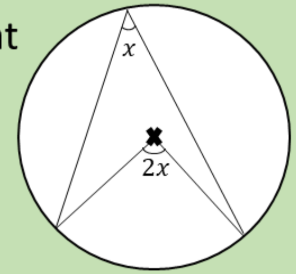
A triangle formed by two radii is an isosceles.



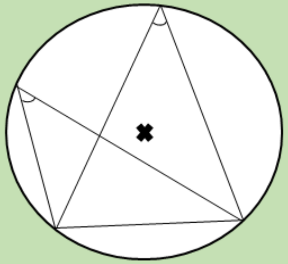
A diameter bisects a chord at right angles, 90^o.



The angle at the centre of the circle is double the angle at the circumference.

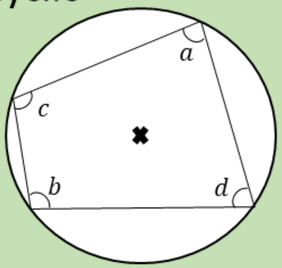


Angles subtended by an arc in the same segment are equal.

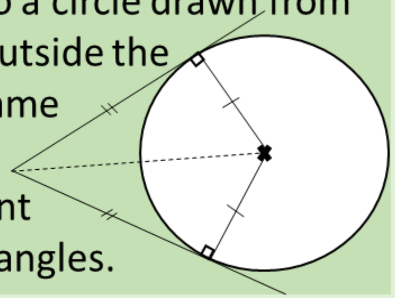


Opposite angles in a cyclic quadrilateral sum to 180^o.

$a + b = 180^o$
$c + d = 180^o$



Two tangents to a circle drawn from a single point outside the circle are the same length and create congruent right-angled triangles.



The angle between a tangent and a chord is equal to the angle subtended from the ends of the chord in the alternate segment. *Alternate Segment Theorem*.