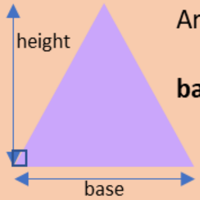
Area and Perimeter

AREA AND PERIMETER – KEY WORDS AND DEFINITIONS	
Area	The number of square units that fit inside the shape.
Perimeter	The total distance around the shape.
Compound shape	A compound shape is made from 2 or more shapes joined together.
Radius	Distance from the centre of the circle to the circumference.
Diameter	Distance from one side of the circle, through the centre, to the other side.
Circumference	The perimeter of a circle.
Dimensions	The measurements of the shape.
Unit of measure	A unit given to tell us the size of the shape. E.g. cm, m, inch, feet, etc.
Sector	A fraction of a circle. It can look like a pizza slice!

Shapes you need to know

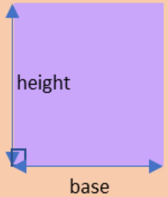
Area formulas



Area of a triangle:

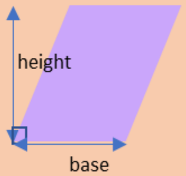
base × height ÷ 2

Height needs to be vertical!



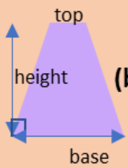
Area of a square or rectangle:

base × height



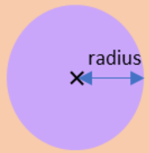
Area of a parallelogram:

base × height



Area of a trapezium:

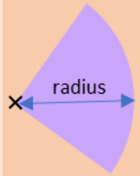
(base + top) × height ÷ 2



Area of a circle:

πr^2

where r is the radius



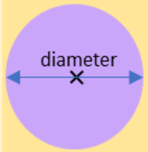
Area of a sector:

$\frac{\theta}{360} \times \pi r^2$

where r is the radius and θ is the angle.

Perimeter

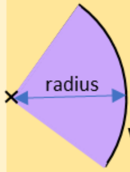
To calculate the perimeter, you find the sum of all the edges of the shape.



Circumference of a circle:

πd

where d is the diameter



Arc length of a sector:

$\frac{\theta}{360} \times \pi d$

where d is the diameter and θ is the angle.

Maths FOUNDATION: Learning Cycle 1

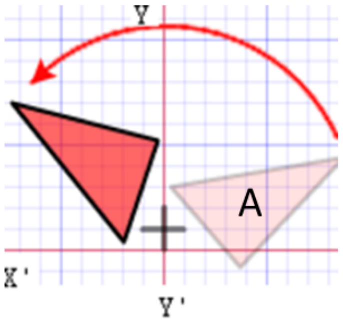
Transformations

TRANSFORMATIONS – KEY WORDS AND DEFINITIONS	
Transformation	The collective name given to the four different methods of changing the position and size of a shape.
Rotation	Turns a shape on a coordinate grid from a given point. The shape does not change size but does change orientation.
Clockwise	Clockwise is the direction a clock turns. ↻
Anticlockwise	Anti-clockwise is the opposite direction. ↺
Centre	The point from where a transformation is measured
Degrees	The unit for measuring angles
Reflection	Creates a mirror image of a shape on a coordinate graph. The mirror line is given by an equation eg. $y = 2$, $x = 2$, $y = x$. The shape does not change in size.
Object	The object is the original shape.
Image	The image is the transformed shape.

Rotate Shape A 90°

anti-clockwise about

(0,1)

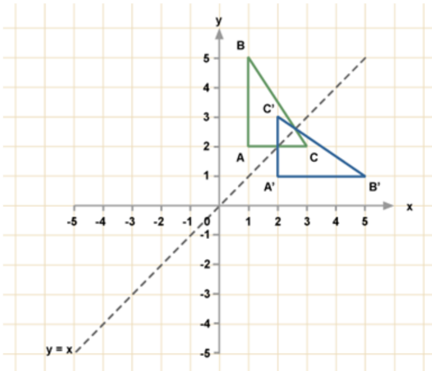


Reflection

The size does not change, but the shape is 'flipped' like in a mirror.

Line $x = ?$ is a vertical line.
Line $y = ?$ is a horizontal line.
Line $y = x$ is a diagonal line.

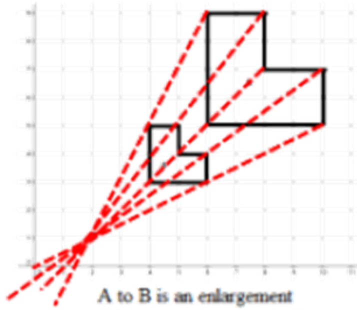
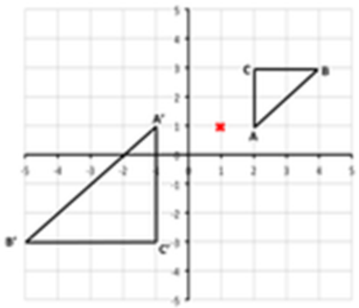
Reflect shape C in the line $y = x$



Transformations

TRANSFORMATIONS – KEY WORDS AND DEFINITIONS	
Enlargement	Enlargement changes the size of an image using a scale factor from a given point.
Positive scale factor	A positive scale factor will increase the size of an image.
Fractional scale factor	A fractional scale factor will reduce the size of an image.
Negative scale factor	A negative scale factor will place the image on the opposite side of the centre of enlargement, with the image inverted.

Scale Factor 3 means 3 times larger
Scale Factor ½ means half the size
Scale Factor -3 means it will be rotated and 3 times bigger

Finding the Centre of Enlargement	**Draw straight lines through corresponding corners of the two shapes.** **The centre of enlargement is the point where all the lines cross over.** **Be careful with negative enlargements as the corresponding corners will be the other way around.**	 A to B is an enlargement SF 2 about the point (2,1)
Negative Scale Factor	Negative enlargements will look like they have been rotated. $SF = -2$ will be rotated, and also twice as big.	Enlarge ABC by scale factor -2, centre (1,1) 

Transformations

TRANSFORMATIONS – KEY WORDS AND DEFINITIONS	
Translate/ Translation	To slide a shape from one position to another using a column vector. Moves a shape on a coordinate grid. Using Column vectors
Column Vector	Used to describe a translation. The top number being how many squares right (+) or left (-). The bottom number is how many square up (+) or down (-). The column vector $\binom{3}{2}$ means 3 right and 2 up. The column vector $\binom{-2}{-2}$ means 2 left and 2 down.

Describing Transformations

Give the following information when describing each transformation:

Look at the number of marks in the question for a hint of how many pieces of information are needed.

If you are asked to describe a 'transformation', you need to say the name of the type of transformation as well as the other details.

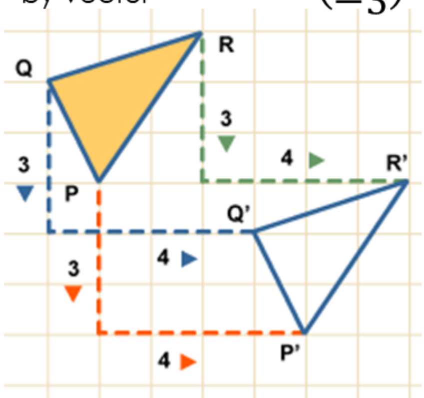
- Translation: Vector

- Rotation: Direction, Angle, Centre

- Reflection: Equation of mirror line

- Enlargement: Scale factor, Centre of enlargement

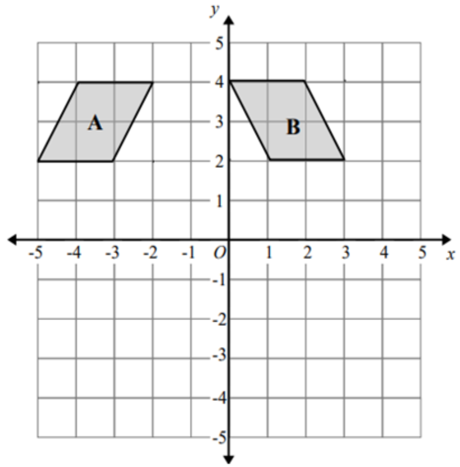
This shape has been translated by vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$



Positive Right $\begin{pmatrix} x \\ y \end{pmatrix}$ Negative Left

Positive Up Negative Down

Describe fully the single transformation that maps shape A onto shape B.



Answer:

Reflection in the line $x = -1$

Rounding and Error Intervals

Topic/Skill	Definition/Tips	Example
Place Value	The value of where a digit is within a number.	In 726, the value of the 2 is 20, as it is in the 'tens' column.
Place Value Columns	The names of the columns that determine the value of each digit. The 'ones' column is also known as the 'units' column.	
Rounding	To make a number simpler but keep its value close to what it was. If the digit to the right of the rounding digit is less than 5, round down. If the digit to the right of the rounding digit is 5 or more, round up.	74 rounded to the nearest ten is 70, because 74 is closer to 70 than 80. 152,879 rounded to the nearest thousand is 153,000.
Decimal Place	The position of a digit to the right of a decimal point.	In the number 0.372, the 7 is in the second decimal place. 0.372 rounded to two decimal places is 0.37, because the 2 tells us to round down. Careful with money - don't write £27.4, instead write £27.40
Significant Figure	The significant figures of a number are the digits which carry meaning (ie. are significant) to the size of the number. The first significant figure of a number cannot be zero. In a number with a decimal, trailing zeros are not significant.	In the number 0.00821, the first significant figure is the 8. In the number 2.740, the 0 is not a significant figure. 0.00821 rounded to 2 significant figures is 0.0082. 19357 rounded to 3 significant figures is 19400. We need to include the two zeros at the end to keep the digits in the same place value columns.
Truncation	A method of approximating a decimal number by dropping all decimal places past a certain point without rounding.	3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416)

Rounding and Error Intervals

Topic/Skill	Definition/Tips	Example
Estimate	To find something close to the correct answer.	An estimate for the height of a man is 1.8 metres.
Approximation	When using approximations to estimate the solution to a calculation, round each number in the calculation to 1 significant figure. ≈ means 'approximately equal to'	$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$ 'Note that dividing by 0.5 is the same as multiplying by 2'

What you need to know: Rounding and Truncation to state error intervals

Key Facts: Rounding a number and truncating are different things. Truncation comes from the word truncare, meaning "to shorten," and can be traced back to the Latin word for the trunk of a tree, which is truncus.

3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416).

A question may ask for the error interval for rounding or truncation – take care to read the question!

The upper and lower bound come from the largest and smallest values that would round to a particular number.

Take 'half a unit above and half a unit below'. For example rounded to 1 d.p means nearest 0.1, so add 0.05 and subtract 0.05 to get the bounds.

All error intervals look the same like this: $\leq x <$

The lowest value a number could have been is the lower bound.

The highest value a number could have been is the upper bound.

E.g. 1 State the upper and lower bound of 360 when it has been rounded to 2 significant figures:

2 significant figures is the nearest 10, so 'half this' to get 5, and add on to 360 and take it off 360, $355 \leq x < 365$

Note: You should know it could be 364.9999… but we write 365 as the upper bound for ease of calculations.

E.g. 2 Truncation: State the error interval of 4.5 when it has been truncated to 1decimal place.

This means it has been 'chopped off'. The lowest value it could have been is 4.5, the highest is 4.59999… so in an error interval
$4.45 \leq x < 4.55$

Quadratics

QUADRATICS – KEY WORDS AND DEFINITIONS	
Quadratic	An expression where the highest power of the variable, usually x, is squared.
Expand	To multiply each term to remove the bracket.
Factorise	The inverse of expanding. To find factors and put into brackets.
Solve	To find the value of the variable, an unknown.
Solution	A value or set of values that satisfy an equation.
Coefficient	The number which the variable is being multiplied by.
Simplify	Group and combine similar terms.
Terms	The result of a single number or variable (or numbers and variables) multiplied together.
Inverse	The opposite operation.

Factorising into a single bracket

Factorise $15x^2 + 3x$

$= 3x(5x + 1)$

Biggest number that goes into 15 and 3

x goes into both terms

What we need to multiply $3x$ by to get the original expression.

You can check your answer by expanding.

Expand Single Brackets

Multiply each term in the bracket by the term on the outside.

Expand and simplify

$3x(x - 4)$

$= 3x^2 - 12x$

Expand Double Brackets

Expand and simplify

$(x + 3)(x + 5)$

$= x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$

First
Outer
Inner
Last

Expand and simplify

$(x - 2)(x + 6)$

$= x^2 - 2x + 6x - 12$

$= x^2 + 4x - 12$

×	x	-2
x	x^2	$-2x$
$+6$	$+6x$	-12

Expand and simplify

$(x + 4)^2$

$= (x + 4)(x + 4)$

$= x^2 + 4x + 4x + 16$

$= x^2 + 8x + 16$

Always write square brackets as two brackets.

Quadratics

Difference of two squares

The difference of two squares is when you have one squared term subtract another squared term.

$$a^2 - b^2 = (a+b)(a-b)$$

e.g. $y^2 - 9 = (y+3)(y-3)$

e.g. $4y^2 - 25 = (2y+5)(2y-5)$

Factorising into double brackets

Steps to factorise quadratic expressions $ax^2 + bx + c$

Step 1 – Write two brackets with x's in: $(x \quad)(x \quad)$

Step 2 – Find two numbers that multiply to give 'c' (the number term) but also add/subtract to give 'b' (the number in front of the x)

Step 3 – Put your numbers in each bracket with the correct $+$ or $-$ signs.

You can always check your answer by expanding.

Factorise $x^2 + 3x + 2$

We need to find two numbers that multiply to make 2, and add or subtract to make 3.

$$= (x+1)(x+2)$$

Factors of 2 that add to make 3.

You can check your answer by expanding.

Factorising into double brackets

Factorise $x^2 - 7x + 12$

We need to find two numbers that multiply to make 12, and add or subtract to make -7.

$$= (x-3)(x-4)$$

Factors of 12 that subtract to make -7.

Factors of 12	
1	12
2	6
3	4

You can check your answer by expanding.

Factorise $x^2 - 5x - 6$

We need to find two numbers that multiply to make -6, and add or subtract to make -5.

$$= (x-6)(x+1)$$

Factors of 6 that add and subtract to make -5.

Factors of 6	
1	6
2	3

You can check your answer by expanding.

Solving quadratics

To solve a quadratic, we need to find the value of x that makes the equation equal zero.
To do this, we factorise as normal then we find the value of x that makes each bracket 0.

Solve $x^2 + 8x - 20 = 0$

We need to find two numbers that multiply to make -20, and add or subtract to make 8.

$$(x+10)(x-2) = 0$$

To make this bracket equal zero, x must be -10.

To make this bracket equal zero, x must be +2.

So the solutions to this equation are

$$x = -10 \text{ and } x = 2$$

Angles

ANGLES – KEY WORDS AND DEFINITIONS

Angle	An **angle** is formed when two lines intersect.
Acute angle	An **acute angle** is less than 90^o.
Right angle	A **right angle** is exactly $\mathbf{90^o}$**.**
Obtuse angle	An **obtuse angle** is greater than 90^o but less than 180^o.
Straight line	A **straight line** has an angle of $\mathbf{180^o}$.
Reflex angle	A **reflex angle** is greater than 180^o but less that 360^o.
Angles around a point.	**Angles around a point** add to make $\mathbf{360^o}$.
Parallel lines	**Parallel lines** never meet and always stay the same distance apart.
Perpendicular lines	**Perpendicular lines** intersect at 90^o.
Degree	**Degrees** are the unit measurement used to measure the size of an angle.
Interior angle	An **interior angle** is an angle on the inside of a shape.
Exterior angle	An **exterior angle** is an angle on the outside of a shape.
Polygon	A **polygon** is a many-sided shape with straight sides.

Types of angles


Acute angle.

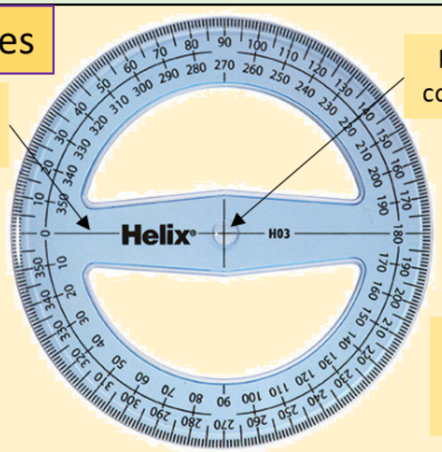

Right angle.


Straight line.


Obtuse angle.

Measuring angles

Place the 0^o line on your angle line.



Place the + on the corner of your angle.

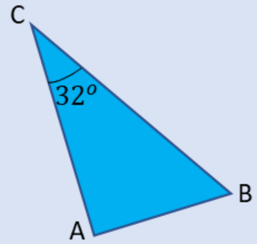
Read the size of your angle from the scale.

Angle notation

We use three letters to name angles. e.g. ACB = 32^o

The middle letter is where the angle is.

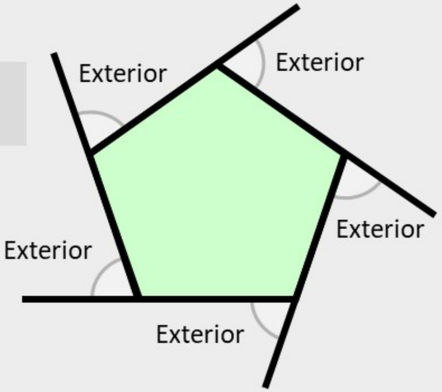
The first and last letter tell you which two lines enclose the angle.



Angles

Exterior angles

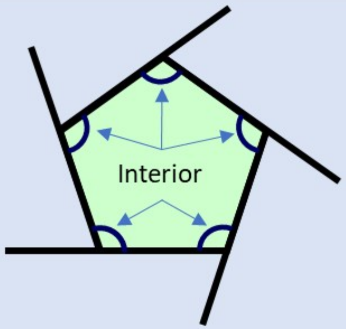
Exterior angles on a polygon sum to 360^o



To calculate one exterior angle on a regular polygon:
360^o ÷ the number of sides

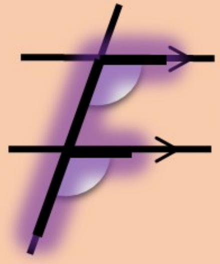
Interior angles

The sum of the interior angles in a polygon:
(number of sides – 2) x 180^o

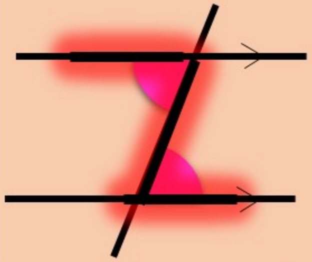


To find one angle in a regular polygon, you divide the sum of the interior angles by the number of sides.

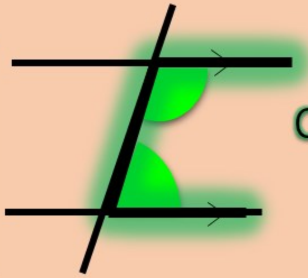
Angles in parallel lines



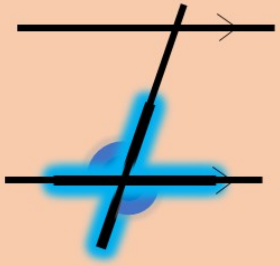
Corresponding angles are equal



Alternate angles are equal



Co-interior angles sum to 180^o



Vertically opposite angles are equal

Indices

INDICES – KEY WORDS AND DEFINITIONS	
Index/exponent/power	An **index**, or a **power**, is the small floating number that goes next to a number or letter.
Indices	**Indices** show how many times a number or letter has been multiplied by itself.
Square number	A number or variable that has been multiplied by itself.
Cube number	A number or variable that has been multiplied by itself and then by itself again.
Square root	The **square root** of a number is the factor that we can multiply by itself to get that number.
Cube root	The **cube root** of a number is the factor that we can multiply by itself and then by itself again to get that number.
Integer	A whole number.
Coefficient	The number which the variable is being multiplied by.
Base number	The number/variable that is being multiplied by itself a given number of times.
Variable	A letter or term that represents an unknown number, value or quantity.

Powers of 10

$10^1 = 10 \quad 10^2 = 100 \quad 10^3 = 1000 \quad 10^4 = 10000$

5^3 — Power — a^2

Base number

Anything to the power of 1 is itself,
e.g. $5^1 = 5, \ \ 123^1 = 123$

Anything to the power of 0 is just 1,
e.g. $6^0 = 1, \ \ 4567^0 = 1$

1 to the power of anything is still 1,
e.g. $1^{10} = 1, \ \ 1^{89} = 1$

$$5^3 = 5 \times 5 \times 5 = 125$$
$$a^2 = a \times a$$

Five rules to remember:

When **multiplying**, you **add** the powers.

e.g. $6^7 \times 6^4 = 6^{7+4} = 6^{11}$

When **dividing**, you **subtract** the powers.

e.g. $x^{19} \div x^{12} = x^{19-12} = x^7$

When raising one power to the another, you **multiply** the powers.

e.g. $(2^5)^8 = 2^{5\times8} = 2^{40}$

When you have a fraction, apply the power to **both** the numerator and denominator.

e.g. $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$

A negative power turns the number **upside-down**.

e.g. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}, \quad \left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{5^2}{2^2} = \frac{25}{4}$

Square roots

Square rooting, $\sqrt{\ }$, is the inverse operation of squaring a number:

e.g. $9^2 = 9 \times 9 = 81$

$\sqrt{81} = 9$

Cube roots

Cube rooting, $\sqrt[3]{\ }$, is the inverse operation of cubing a number:

e.g. $2^3 = 2 \times 2 \times 2 = 8$

$\sqrt[3]{8} = 2$

Fractions

FRACTIONS – KEY WORDS AND DEFINITIONS	
Fraction	Describes parts of a whole.
Vinculum	The line between the number at the top and bottom of the fraction.
Numerator	The number on top of the fraction.
Denominator	The number at the bottom of the fraction.
Proper fraction	A fraction where the numerator is smaller than the denominator.
Improper fraction	A fraction where the numerator is larger than the denominator.
Mixed number	A number composed of a whole number and a fraction.
Equivalent	Two or more fractions that express the same part of a whole. There is a number by which both the numerator and denominator of one fraction can be multiplied or divided to get an equivalent fraction.
Simplify	The act of dividing the numerator and denominator by a common factor until there are no more common factors to divide by.

Fraction facts

Numerator → 3

Vinculum → the line

Denominator → 4

$$\frac{3}{4}$$

Mixed number	Proper fraction	Improper fraction
$4\frac{2}{5}$	$\frac{7}{10}$	$\frac{11}{5}$

Simplifying fractions

To simplify fractions, we find a common factor of the numerator and the denominator and divide. We continue dividing until there are no more common factors.

Simplify $\frac{30}{42}$

30 and 42 are both even so divide by 2.

$$\frac{30}{42} \xrightarrow{\div 2} \frac{15}{21} \xrightarrow{\div 3} \frac{5}{7}$$

If you find the highest common factor, 6, you can divide by that straight away.

Mixed numbers

You need to able to convert between mixed numbers and improper fractions.

Mixed numbers → Improper fractions

Step 1: Multiply the whole number by the numerator.

Step 2: Add the numerator to the answer to step 1.

Step 3: Write the answer to step 2 on the numerator and keep the denominator the same as the question.

$$6\frac{3}{4} = \frac{(6 \times 4) + 3}{4} = \frac{24 + 3}{4} = \frac{27}{4}$$

Improper fractions → Mixed numbers

Step 1: Work out how many times the denominator goes into the numerator. This is your whole number.

Step 2: Work out the remainder needed to get the numerator. This is your new numerator. Keep the denominator the same as in the question.

5,10,15,20,...
3 whole 5s go into 19.
The remainder is 4.

$$\frac{19}{5} = 3\frac{4}{5}$$

Fractions

Multiplying fractions

Multiply together your numerators, followed by your denominators. Then simplify if possible.

This is the easiest fraction calculation!

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

Dividing fractions

Keep

Flip

Change

Keep your first fraction the same.

Flip your second fraction over.

Change the sign from a divide to a multiply.

$$\frac{4}{7} \div \frac{2}{5} = \frac{4}{7} \times \frac{5}{2} = \frac{4 \times 5}{7 \times 2} = \frac{20}{14} = 1\frac{6}{14} = 1\frac{3}{7}$$

Adding fractions

Convert mixed numbers into improper fractions!

To add fractions, you need to have a common denominator.

If your fractions have different denominators, you will need to change the denominators first by finding a common multiple.

Once the denominators are the same, you just add the numerators together and simplify if possible.

Same denominators

$$\frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$$

Different denominators

30 is the LCM of 10 and 6.

$\times 3$ $\quad \times 5$

$$\frac{3}{10} + \frac{5}{6} = \frac{9}{30} + \frac{25}{30} = \frac{34}{30} = 1\frac{4}{30} = 1\frac{2}{15}$$

Subtracting fractions

Convert mixed numbers into improper fractions!

To subtract fractions, you follow the same initial steps as for adding. You then subtract your fractions instead of adding.

Same denominators

$$\frac{4}{8} - \frac{3}{8} = \frac{1}{8}$$

Different denominators

$\times 5$ $\quad \times 7$

$$4\frac{3}{7} - 2\frac{4}{5} = \frac{31}{7} - \frac{14}{5} = \frac{155}{35} - \frac{98}{35} = \frac{57}{35} = 1\frac{22}{35}$$

35 is the LCM of 7 and 5.

Convert back into a mixed number.

Fractions of an amount

To calculate a fraction of an amount, you divide by the denominator and multiply by the numerator.

Calculate $\frac{4}{5}$ of 35

Divide by the denominator $35 \div 5 = 7$

Multiply by the numerator $4 \times 7 = 28$

So $\frac{4}{5}$ of 35 = 28

Expressing as a fraction

To express a number as a fraction of another number, you write the first number as the numerator and the second number as the denominator. Simplify if possible.

Write 42 as a fraction of 50.

$$\frac{42}{50} \overset{\div 2}{\underset{\div 2}{=}} \frac{21}{25}$$